PERGAMON

International Journal of Solids and Structures 40 (2003) 5187–5196

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

www.elsevier.com/locate/ijsolstr

Stability and stabilization of circular plate parametric vibrations

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Received 7 October 2002; received in revised form 24 January 2003

Abstract

The stability of parametric vibrations of circular plate subjected to in-plane forces is analyzed by the Liapunov method. Assuming that the compressing forces are physically realizable ergodic processes the plate dynamics is described by stochastic classical partial differential equations. The energy-like functional is proposed; its positiveness is equivalent to the condition in which static buckling does not occur. Taking into account that a plate is compressed radially by time-dependent and uniformly distributed along its edge forces, a dynamic stability of an undeflected state of isotropic elastic circular plate is analyzed. The rate velocity feedback is applied to stabilize the plate parametric vibration. The critical damping coefficient has been expressed by the variance and the mean value of compressing force. The admissible variances of loading strongly depend on the feedback gain factor.

Keywords: Dynamic stability; Circular plate; Active stabilization; Piezoelectric layers

1. Introduction

Axisymmetric (rotating or not) disks are common elements in modern structures and machinery, such as diaphragms and shields in acoustic ducts, turbine disks, circular saws, and memory disk units. Timoshenko (1936) calculated the critical compressive force uniformly distributed on the edge of elastic circular plate for different boundary conditions. The first purely analytical analysis of elastic rotating disks was by Lamb and Southwell (1921). Mote (1965) analyzed free vibrations of initially prestressed circular disks. Mostaghel and Tadjbakhsh (1973) studied the eigenvalue problem for rotating elastic circular plate, and devised numerical procedures to evaluate the critical rotation speed and its dependence on the relevant parameters. Seubert et al. (2000) investigated constrained layer damping in computer disk drives. Applying the finite element method they calculated natural frequencies and damping ratios for the damped disks.

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doi:10.1016/S0020-7683(03)00263-4

The result provide damping values useful in the stability analysis of spinning disks that include the passive damping. Auburn et al. (1984) presented the detailed approach and experimental results for controlling both the rigid body and five structural bending modes of a circular plate used in large space structures. Kuo and Huang (1992) developed analysis and simulation of a control system design method which can stabilize all the vibration modes of a flexible distributed parameter system with a discrete set of sensors and actuators without involving truncation of the higher frequency modes. The method based on the root locus arguments for infinite dimensional systems was applied to a thin rotating with constant angular velocity centrally clamped circular disk. Manzione and Nayfeh (2001a,b) analyzed the transverse vibrations of a circular disk of uniform thickness rotating about its axis with constant angular velocity, when the disk is subject to a space-fixed spring-mass-dashpot system. Using the method of multiple scales they determined a nonlinear system of equations describing the modulation of the amplitudes and phases of two interacting modes. The authors obtained interacting solutions among lowest modes and stability characteristics of these solutions.

Piezoelectric materials show great advantages as actuators in intelligent structures i.e. structures with highly distributed actuators, sensors, and processor networks. Piezoelectric sensors and actuators have been applied successfully in the closed loop control (Bailey and Hubbard, 1985). Crawley and de Luis (1987) presented a comprehensive static model for a piezoelectric actuator glued to a beam. The relationship between static structural strains, both in the structure and in the actuator, and the applied voltage across the piezoelectric was presented. This static approach was then used to predict the dynamic behavior. The direct Liapunov method was applied to the stabilization problem of the beam subjected to time-dependent axial forces (Tylikowski, 1993).

An active control approach that reduces transient noise transmission through a membrane placed in a circular duct was presented by van Niekerk and Tongue (1995). Different control strategies were investigated analytically and than implemented experimentally. Structural vibration of a circular plate due to the excitation of a piezoelectric circular actuator was modeled by van Niekerk et al. (1995) using a static approach to modeling the actuator–plate interaction. The dynamic stability of circular plates uniformly compressed by broad-band radial forces and described by stochastic Itô equations was investigated analytically by Tylikowski and Frischmuth (2001). The stability domains have been expressed by the intensity and the mean value of the compressing force and the critical damping coefficients. A dynamic model for the simply supported circular plate with a piezoelectric actuator glued to each of its upper and lower surfaces was developed by Tylikowski (1999a). In the model the actuators were assumed to be perfectly bonded. It means that the bonding layer is sufficiently thin that the shear of layer can be neglected. Vibrations of capacitively shunting distributed piezoelectric elements perfectly glued to the vibrating annular plate excited by harmonic displacement of the inner edge was analyzed by Tylikowski (2001).

The aim of the present analytical work is to generalize the previous result concerning beams—one dimensional continuous systems (Tylikowski, 1999b) and to investigate an asymptotic stability of circular plate compressed radially by time-dependent forces. The problem is reduced to the partial differential equation of the three independent variables with time-dependent coefficient. The stability of parametric vibrations of circular plate subjected to in-plane forces is analyzed by the Liapunov method. Assuming that the compressing forces are physically realizable ergodic processes the plate dynamics is described by a stochastic classical partial differential equation. The energy-like functional is proposed; its positiveness is equivalent to the condition in which static buckling does not occur. Taking into account that a plate is compressed radially by time-dependent and uniformly distributed along its edge forces, a dynamic stability of an undeflected state of isotropic elastic circular plate is analyzed. The rate velocity feedback is applied to stabilize the plate parametric vibration. The critical damping coefficient has been expressed by the variance and the mean value of compressing force. The admissible variances of loading strongly depend on the feedback gain factor.

2. Problem formulation

We start with the problem of dynamic stability of circular plate without the active vibration control. Consider transverse vibrations of a thin elastic circular plate clamped on its edge. The Kirchhoff hypothesis on nondeformable normal elements to the middle plane is used and the rotary and coupling inertias are neglected. Taking into account the in-plane compressive forces, we assume that they are uniformly distributed along the plate circular edge of radius R. The forces are time-dependent and consist of a constant mean value S_0 and time-dependent oscillating part $S(t)$. The thickness of the plate t_p is constant and the energy of the transverse motion is dissipated by a viscous damping with the constant proportionality coefficient β. The mass density is denoted by ρ and the bending stiffness by D. The dynamic equation of the transverse plate motion has the classical form

$$\rho t_\mathrm{p}\frac{\partial^2 w}{\partial t^2} + 2\beta\rho t_\mathrm{p}\frac{\partial w}{\partial t} + D\Delta^2 w + (S_0 + S(t))\Delta w = 0 \quad (r,\varphi) \in \Omega \equiv \{0,R\} \times \{0,2\Pi\} \tag{1}$$

where

$$\Delta = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2}$$

Boundary conditions corresponding to clamped edge are as follows

$$w(R) = 0 \quad \frac{\partial w}{\partial r}(R) = 0 \tag{2}$$

Dividing by ρt_p we introduce the notations

$$e = \frac{D}{\rho t_\mathrm{p}} \quad f_0 = \frac{S_0}{\rho t_\mathrm{p}} \quad f(t) = \frac{S(t)}{\rho t_\mathrm{p}}$$

and rewrite Eq. (1) in the form

$$\frac{\partial^2 w}{\partial t^2} + 2\beta\frac{\partial w}{\partial t} + e\Delta^2 w + (f_0 + f(t))\Delta w = 0 \quad (r,\varphi) \in \Omega \tag{3}$$

The main purpose of the paper is to examine an almost sure asymptotic stability of the plate equilibrium state. To estimate a perturbed solution of Eq. (3) it is necessary to introduce a measure of distance $\|\ \|$ of the solution of Eq. (3) with nontrivial initial conditions from the trivial one. The equilibrium state of Eq. (3) is called almost sure asymptotically stable if

$$\lim_{t\to\infty} P\{\|w(.,t)\| = 0\} = 1 \tag{4}$$

where P denotes the probability measure. In the present paper we shall use the Liapunov technique derived for the Euler–Bernoulli beam by Kozin (1972), which provides a significant advantage in that the conditions for stability can be obtained without explicitly solving the plate equation of motion. The method is used to establish criteria for the almost sure stochastic stability of the unperturbed (trivial) solution of the plate compressed by the uniformly distributed time-dependent forces.

3. Dynamic stability analysis

If the parametric excitation is a physically realizable ergodic process, Eq. (3) is understood as the partial differential equation with a random parameter. In order to examine the uniform stability of the plate equilibrium (the trivial solution $w = 0$), we choose the Liapunov functional as a sum of the modified kinetic energy and the elastic energy of the plate (Tylikowski, 1993)

$$V = \frac{1}{2}\int_0^R \int_0^{2\Pi} \left[v^2 + 2\beta v w + 2\beta^2 + e\left[(\Delta w)^2 - 2(1-\nu)\frac{\partial^2 w}{\partial r^2}\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \varphi^2}\right) + 2(1-\nu)\left(\frac{1}{r}\frac{\partial^2 w}{\partial r \partial \varphi} - \frac{1}{r^2}\frac{\partial w}{\partial \varphi}\right)^2 \right] + f_0 w \Delta w \right] r\,\mathrm{d}r\,\mathrm{d}\varphi \tag{5}$$

where v is the velocity of transverse motion. The classical static buckling loading f_{cr} can be calculated from variational inequality

$$\int_0^R \int_0^{2\Pi} \left[(\Delta w)^2 - 2(1-\nu)\frac{\partial^2 w}{\partial r^2}\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \varphi^2}\right) + 2(1-\nu)\left(\frac{1}{r}\frac{\partial^2 w}{\partial r \partial \varphi} - \frac{1}{r^2}\frac{\partial w}{\partial \varphi}\right)^2 \right] r\,\mathrm{d}r\,\mathrm{d}\varphi \geqslant f_{\mathrm{cr}} \int_0^R \int_0^{2\Pi} \left[\left(\frac{\partial w}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial w}{\partial \varphi}\right)^2 \right] r\,\mathrm{d}r\,\mathrm{d}\varphi \tag{6}$$

If the static buckling condition is fulfilled ($f_0 < f_{\mathrm{cr}}$), inequality (6) holds and therefore functional (5) is positive-definite. Therefore, the measure of distance between the perturbed solution and the trivial one can be chosen as the square root of the functional $\| \; \| = V^{1/2}$. Upon differentiation with respect to time and using the boundary conditions (2) we obtain the time-derivative of functional (5) in the form

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -2\beta V + 2U \tag{7}$$

where

$$U = \frac{1}{2}\int_0^R \int_0^{2\Pi} \left(2\beta^2 v w + 2\beta^3 w^2 - (w + \beta w) f(t) \Delta w \right) r\,\mathrm{d}r\,\mathrm{d}\varphi \tag{8}$$

We look for a function λ defined as a maximum over all admissible functions w and v satisfying the boundary conditions of the ratio U/V

$$\lambda = \underbrace{\max}_{w,v} \frac{U(w,v)}{V(w,v)} \tag{9}$$

Because a maximum is a particular case of a stationary point, we put to zero a variation of U/V and obtain the variational equation

$$\delta(U - \lambda V) = 0 \tag{10}$$

The appropriate Euler equations have the form

$$\begin{aligned} &\beta(\beta-\lambda)w - \lambda v - \frac{f(t)}{2}\Delta w = 0 \\ &\beta(\beta-\lambda)(v + 2\beta w) - \frac{f(t)}{2}(2\beta\Delta w + \Delta v) - \lambda f_0 \Delta w - \lambda e \Delta^2 w = 0 \end{aligned} \tag{11}$$

Using properties of Bessel functions we can prove that for solutions of boundary problem (2) represented as

$$w_{mn}(r,\varphi) = [J_n(\kappa_{mn} r) + C_{mn} I_n(\kappa_{mn} r)](\sin n\varphi + \cos n\varphi) \tag{12}$$

where κ_{mn} is the wavenumber, $m = 1, 2, 3, \ldots$ denotes the number of nodal circles and $n = 0, 1, 2, \ldots$ denotes the number of nodal diameters, C_{mn} are the known constants, J_n and I_n are the Bessel functions

$$\Delta w_{mn} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w_{mn}}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 w_{mn}}{\partial \varphi^2} = -\kappa_{mn}^2 w_{mn} \tag{13}$$

Eliminating the operator Δ by means of (13) we solve the Euler equations with boundary conditions (2) and finally we have

$$\lambda_{mn} = \frac{|\beta^2 + \kappa_{mn}^2 f(t)/2|}{\sqrt{(e\kappa_{mn}^2 - f_0)\kappa_{mn}^2 + \beta^2}} \tag{14}$$

Therefore, in the class of functions (12) the following inequality holds for any pair of indices

$$U(w_{mn}, v_{mn}) \leqslant \lambda_{mn} V(w_{mn}, v_{mn}) \tag{15}$$

We have the following chain of inequalities

$$U = \sum_{m=1}^{\infty}\sum_{n=0}^{\infty} U(w_{mn}, v_{mn}) \leqslant \sum_{m=1}^{\infty}\sum_{n=0}^{\infty} \lambda_{mn} V(w_{mn}, v_{mn}) \leqslant \underbrace{\overbrace{\max}_{m=1,2,3,\ldots}}_{n=0,1,2,\ldots} \lambda_{mn} \sum_{m=1}^{\infty}\sum_{n=0}^{\infty} V(w_{mn}, v_{mn}) = \lambda V \tag{16}$$

where

$$\lambda = \underbrace{\overbrace{\max}_{m=1,2,3,\ldots}}_{n=0,1,2,3,\ldots} \frac{|\beta^2 + \kappa_{mn}^2 f(t)/2|}{\sqrt{(e\kappa_{mn}^2 - f_0)\kappa_{mn}^2 + \beta^2}} \tag{17}$$

Using the basic inequality (16) we can estimate the time-derivative of functional (5) as follows

$$\frac{dV}{dt} \leqslant -2(\beta - \lambda)V \tag{18}$$

The first order differential inequality (18) has the following solution

$$V(t) \leqslant V(0)\exp\left[-2\left(\beta - \frac{1}{t}\int_0^t \lambda(\tau, \omega)\,d\tau\right)t\right] \tag{19}$$

where ω is an element of probability space. As the function λ is the known function of in-plane forces Eq. (17) the ergodicity of the in-plane forces implies the ergodicity of function λ. It means that

$$E\lambda = \lim_{t\to\infty} \frac{1}{t}\int_0^t \lambda(\tau, \omega)\,d\tau \tag{20}$$

where E denotes the mathematical expectation (averaging over the probability space).

Therefore, using the ergodicity of the in-plane forces we can state that the trivial solution of Eq. (3) is almost sure asymptotically stable if

$$\beta \geqslant E(\lambda) \tag{21}$$

Inequality (21) gives us a possibility to obtain minimal damping coefficient guaranteeing the almost sure asymptotic stability called critical damping coefficients. A domain where damping coefficients are greater that the critical damping coefficient is called the stability region. The stability regions as functions of loading variance, damping coefficient, constant component of in-plane force are calculated numerically using an approximate method. First, discrete values of force f are chosen, the largest value λ is determined, and the expectation is calculated numerically integrating the product of λ by the probability density

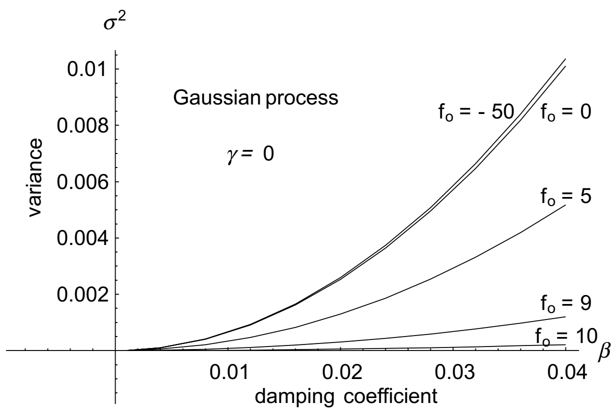

Fig. 1. Stability domains of the circular plate for the Gaussian force.

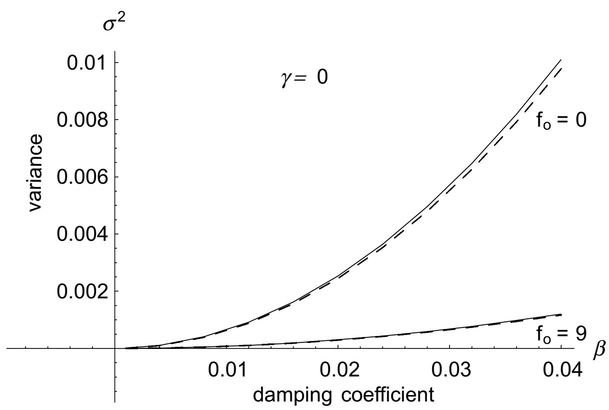

Fig. 2. Comparison of stability domains of the circular plate for the Gaussian (continuous line) and harmonic force (dotted line).

function of in-plane loading. This is accomplished for various values of parameters by choosing the variance and varying the damping coefficient until inequality (21) will be satisfied. Numerical calculations are performed for the Gaussian process with the constant mean value f_0 and variance σ^2 and for the harmonic process with an amplitude A. In order to compare both processes the variance of harmonic process $\sigma^2 = A^2/2$ is used. Fig. 1 shows the influence of constant component of in-plane force on stability region for the Gaussian parametric excitation. The coefficient $\gamma = 0$ denotes that the plate without the active vibration control is analyzed. In Fig. 2 a comparison of stability regions for the Gaussian and the harmonic process. The stability regions are similar, but the Gaussian loading needs smaller critical damping coefficient than the harmonic loading.

4. Plate dynamics equation with distributed feedback

Consider the circular plate with piezoelectric layers of thickness t_s mounted on each of two opposite plate sides. The layers are polarized in the direction perpendicular to the plate. The sensing and actuating effects of piezoelectric layers are used to stabilize both the free vibration due to initial disturbances and the parametric vibration excited by the in-plane time-dependent forces. The sensor electric displacement is given by

$$D_3 = \varepsilon_r e_{3r} + \varepsilon_t e_{3t} \tag{22}$$

where e_{3r} and e_{3t} are piezoelectric constants and ε_r, ε_t are the radial and circumferential strains of sensor, respectively given by

$$\varepsilon_r = -\frac{t_p + 2t_s}{2}\frac{\partial^2 w}{\partial r^2} \tag{23}$$

$$\varepsilon_t = -\frac{t_p + 2t_s}{2}\frac{1}{r}\left(\frac{1}{r}\frac{\partial^2 w}{\partial \varphi} + \frac{\partial w}{\partial r}\right) \tag{24}$$

The electric charge is obtained by integrating the electric displacement over the sensor surface assuming the appropriate sensor polarization profile Φ^s. It is assumed that the piezoelectrical properties are axially symmetric, therefore $e_{3r} = e_{3t} = e_{31}$

$$Q_s = -\frac{e_{31}(t_p + 2t_s)}{2}\int_0^R \int_0^{2\Pi} \Delta w \Phi^s r \,\mathrm{d}r \,\mathrm{d}\varphi \tag{25}$$

Using the formula for a flat capacitor we obtain the sensor voltage V_s

$$V_s = -\frac{e_{31}(t_p + 2t_s)t_s}{4\epsilon_{33}A_s}\int_0^R \int_0^{2\Pi} \Delta w \Phi^s r \,\mathrm{d}r \,\mathrm{d}\varphi \tag{26}$$

where ϵ_{33} is the dielectric constant, A_s is the sensor surface.

Assuming the velocity feedback control the voltage applied to the actuator is

$$V_a = K_a \frac{\mathrm{d}V_s}{\mathrm{d}t} \tag{27}$$

where K_a is the gain factor.

Actuator stresses are equal

$$\sigma_{ar} = \frac{d_{a31} V_a \Phi^a}{t_a} \tag{28}$$

$$\sigma_{at} = \frac{d_{a32} V_a \Phi^a}{t_a} \tag{29}$$

where d_{a31} and d_{a32} are the piezoelectric strain/charge coefficients of actuator. Φ^a is the axially symmetric actuator polarization profile, i.e. $\Phi^a = \Phi(r)$. Integrating the actuator stresses with respect to the coordinate perpendicular to the plate yields the equivalent electric moments

$$M_r^e = \sigma_{ar} t_a \frac{t_p + t_a}{2} \tag{30}$$

$$M_\varphi^e = \sigma_{at} t_a \frac{t_p + t_a}{2} \tag{31}$$

We modify the dynamic equation (3) introducing the axially symmetric bending moments of electric origin

$$\frac{\partial^2 w}{\partial t^2} + 2\beta\frac{\partial w}{\partial t} + e\Delta^2 w + (f_0 + f(t))\Delta w + \frac{\partial^2 m_r^e}{\partial r^2} + \frac{2}{r}\frac{\partial m_r^e}{\partial r} - \frac{1}{r}\frac{\partial m_\varphi^e}{\partial r} = 0 \quad (r,\varphi) \in \Omega \tag{32}$$

where the moments of electric origin m_r^e and m_φ^e are defined as follows

$$m_r^e = \frac{M_r^e}{\rho t_p} = -\frac{(t_p + t_a)(t_p + 2t_s)t_a t_s e_{31} d_{a31} K_a}{8\rho t_p A_s \epsilon_{33}} \Phi^a \int_0^R \int_0^{2\Pi} \Delta w \Phi^s r \, dr \, d\varphi \tag{33}$$

$$m_\varphi^e = \frac{M_r^e}{\rho t_p} = -\frac{(t_p + t_a)(t_p + 2t_s)t_a t_s e_{32} d_{a31} K_a}{8\rho t_p A_s \epsilon_{33}} \Phi^a \int_0^R \int_0^{2\Pi} \Delta w \Phi^s r \, dr \, d\varphi \tag{34}$$

Substituting expressions (33) and (34) into (32) we obtain the equation of circular plate with distributed velocity feedback. The equation is linear with the trivial solution $w = 0$ (equilibrium state).

5. Stabilization of plate parametric vibrations

The introductory stability problem being solved we can direct our attention to the stability analysis of the plate with active vibration control. We assume the same Liapunov functional in the form (5). Calculating the time-derivative of the functional we take into account a modification of dynamic equation introduced by the electric moments. Assuming the polarization profiles of the sensor and actuator in the form of the first axially symmetric mode we repeat the derivation of the function λ. The total feedback gain factor of modal control is denoted by γ. The function λ is defined in the following way

$$\lambda = \underbrace{\underbrace{\max}_{m=1,2,3,\dots}}_{n=0,1,2,\dots} \lambda_{mn} \tag{35}$$

where λ_{mn} for $m, n = 1, 2, 3, \dots$ are defined in the same way as in the stability problem Eq. (14) and λ_{10} depends on the feedback control and is given by

$$\lambda_{10} = \sqrt{\gamma^2 + \frac{(\beta^2 + \kappa_{10}^2 f(t)/2)^2}{(e\kappa_{10}^2 - f_0)\kappa_{10}^2 + \beta^2}} - \gamma \tag{36}$$

The stability regions as functions of loading variance σ^2, damping coefficient, feedback gain factor γ, and constant component of in-plane force are calculated numerically using an iterative method since the inequality (21) defining the stability regions is transcendental. Fig. 3 shows the comparison of stability do-

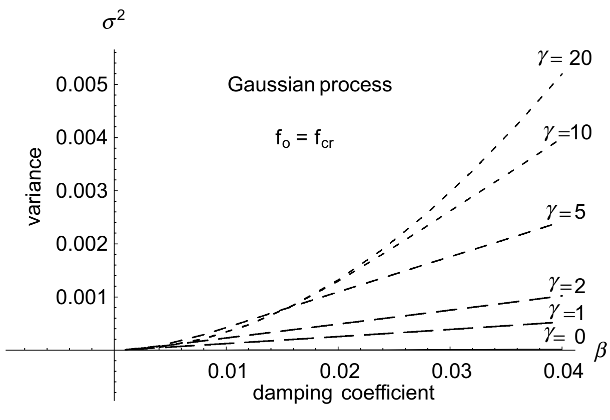

Fig. 3. Stability domains of the circular plate for the critical Gaussian force with single-mode control.

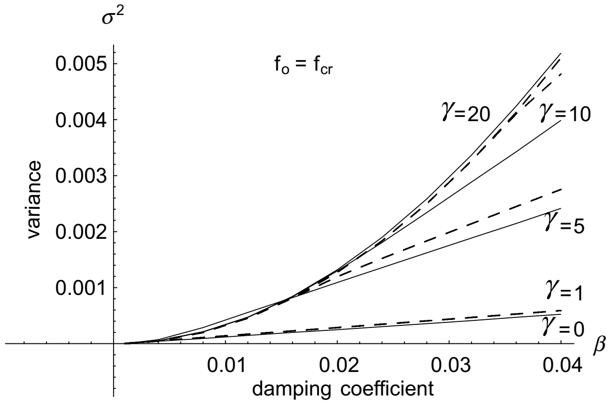

Fig. 4. Comparison of stability domains of the circular plate for the Gaussian and harmonic force with single-mode control.

mains for the critical mean force with a single-mode velocity feedback. It is seen that the stability regions enlarge as the gain factor increases. The active vibration control stabilizes the plate compressed by the critical force f_{cr}. As long as the control is not active ($\gamma = 0$) the stability region disappears. The influence of the feedback gain is small for stretching mean forces. A saturation effect on stability regions is observed for large values of gain factor. Fig. 4 compares the stability regions for the plate with critical mean force loaded by the Gaussian force (continuous line) and harmonic force (dotted line), respectively. It is visible that the influence of the class of excitation is noticeable for $3 \leqslant \gamma \leqslant 10$.

6. Conclusions

By means of the direct Liapunov method the influence of active stabilization of a parametrically excited circular plate with distributed piezoelectric sensor and actuator, and the velocity feedback has been studied. The plate is clamped and subject to axially symmetric in-plane forces randomly fluctuating. Without any passive damping and control the plate vibrations are unstable due to the parametric excitation. The stabilization of stochastic vibrations needs a sufficiently large active damping coefficient. A saturation effect of influence of gain factor on stability domains is observed. The stability domains do not change qualitatively when going from the Gaussian process to the harmonic one.

Acknowledgement

This study was supported by the Polish State Committee for Scientific Research (Grant BS—No 504/G/1152).

References

Auburn, J.N., Ratner, M.J., Lyons, M.G., 1984. Structural control for a circular plate. AIAA J. Guidance, Control and Dynamics 7, 535–545.

Bailey, T., Hubbard, J.E., 1985. Distributed piezoelectric-polymer active vibration control of a cantilever beam. J. Guidance, Control and Dynamics 8, 605–611.

Crawley, E.F., de Luis, J., 1987. Use of piezoelectric actuators as elements of intelligent structures. AIAA J. 25, 1373–1385.
Kozin, F., 1972. Stability of the linear stochastic system. Lecture Notes in Mathematics 294, 186–229.
Kuo, C.Y., Huang, C.C., 1992. Active control of mechanical vibrations in a circular disk. ASME J. Dynamic Systems, Measurements and Control 114, 104–112.
Lamb, H., Southwell, R., 1921. The vibrations of spinning disks. Proc. Royal Soc. London 99, 272–280.
Manzione, P., Nayfeh, A.H., 2001a. Combination resonance of a centrally clamped rotating circular disk. J. Vibration and Control 7, 979–1011.
Manzione, P., Nayfeh, A.H., 2001b. Instability mechanisms of a centrally clamped rotating circular disk. J. Vibration and Control 7, 1013–1034.
Mostaghel, N., Tadjbakhsh, I., 1973. Buckling of rotating rods and plates. Int. J. Mech. Sci. 15, 429–434.
Mote Jr., C.D., 1965. Free vibrations of initially stressed circular disks. ASME J. Engng. for Industry 87, 258–264.
Niekerk, J.L., Tongue, B.H., 1995. Active control of a circular membrane to reduce transient noise transmission. Trans. ASME, J. Vibration and Acoustics 117, 252–258.
Niekerk, J.L., Tongue, B.H., Packard, J.R., 1995. Active control of a circular plate to reduce transient noise transmission. J. Sound and Vibration 183, 643–662.
Seubert, S.L., Anderson, T.J., Smelser, R.E., 2000. Passive damping of spinning disk. J. Vibration and Control 6, 715–725.
Timoshenko, S., 1936. Theory of Elastic Stability. McGraw-Hill, New York, London.
Tylikowski, A., 1993. Stabilization of beam parametric vibrations. J. Theoretical and Appl. Mech. 31, 657–670.
Tylikowski, A., 1999a. Simulation examination of annular plates excited by piezoelectric actuators. In: Smart Structures. Kluwer, Dordrecht, pp. 365–372.
Tylikowski, A., 1999b. Stabilization of beam parametric vibrations by means of distributed piezoelectric elements. J. Theoretical and Appl. Mech. 37, 241–254.
Tylikowski, A., 2001. Control of circular plate vibrations shunted with a capacitive circuit. Thin-Walled Structures 39, 83–94.
Tylikowski, A., Frischmuth, K., 2001. Dynamic stability of circular plates. In: Proceedings of DINAME IX. The Brazilian Society Mechanical Sciences, Santa Catarina, pp. 335–340.